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THE ABSENCE OF STATIC, SMOOTH SOLUTIONS IN EINSTEIN-YANG-MILLS-KLEIN-GORDON THEORY

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We study the existence of smooth, static nontrivial solutions to Einstein-Yang-Mills-Klein-Gordon equations. The absence of static solutions is proven if the Klein-Gordon field is linear and the asymptotic falloff of g_{00} to unity is quicker than $1/r$. In the case when $g_{00} = 1+O\left(\frac{1}{r}\right)$ the system is shown to reduce to the pure gravity, under certain conditions. Possible applications of bifurcation theory for finding solutions which are close to the trivial one in the case when the scalar field is of Higgs type are discussed.

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1. *Introduction*

The first nonexistence result in the sourceless Yang-Mills theory is due to Deser [1], who has shown that there are no nonzero static finite energy solutions. Then many authors [2, 3] proved nonexistence of time-dependent solitons in Yang-Mills theory. Glassey and Strauss [3] demonstrated the absence of static solitons and solitary waves in Yang-Mills-Klein-Gordon theory. Recently Weder [4] proved the absence of nonsingular, localized solutions to Einstein-Yang-Mills equations. His result needed a strong falloff of g_{00} at infinity and therefore met a critique [5]. Deser [5] attacked the same problem in $(2+1)$ space-time dimensions, to get the desired nonexistence result. Finally in [6] it was shown that the linear scalar (Schrödinger or Klein-Gordon) and nonlinear (under simple restrictions on the nonlinearities) field coupled to Yang-Mills fields has no static nonzero finite energy solutions.

In what follows we continue this line of research and consider the gravitation-nonabelian gauge-scalar system. We give, following the ideas outlined in [6], a simple nonexistence proof of localized static finite energy solutions, supposing that $g_{00} = 1-M/r+o(r^{-1})$, where M is either zero or negative. This is done in Section 2. We state this result for both

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linear and nonlinear scalar field under certain restriction imposed on the scalar selfinteraction term.

In Section 3 we consider the case when the asymptotic conditions on g_{00} are relaxed; they correspond to the nonzero positive gravity mass. As well as nonexistence of nonzero scalar fields we prove the absence of gauge fields sufficiently small in the norm of Sobolev space $W_{1,3}$, provided that the L_3 norm of the derivatives of the determinant $\sqrt{-g}$ is sufficiently small.

Section 4 is devoted to a local analysis of the existence problem. We briefly discuss the possible applications of various methods of bifurcation theory for finding those solutions which are close to the trivial one, in the case when the scalar field is of Higgs type.

2. *Statement of the model*

Let the gauge group be any compact semisimple Lie group G. Consider the G-algebra valued connection one-form A and its curvature two-form F. In local coordinates the curvature (a Yang-Mills strength field) tensor is defined as below

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu, \tag{1}$$

where f^{abc} are the structure constants of the Lie algebra $\mathbf{G}$ of the group G and g is the coupling constant. The structure constants can be chosen to be completely antisymmetric, because of compactness of G. The Greek space-time indices range from 0 to 3 while the latin (algebra) $a, b, c, \ldots$ and (space) $i, j, k, \ldots$ indices range from 1 to the dimension of the algebra and from 1 to 3, respectively.

Let the symbols $\{\varphi^a\}$ designate the set of components of the scalar Klein-Gordon field. The doubly covariant derivative is

$$D_\mu = \nabla_\mu + g[A_\mu, \cdot], \tag{2}$$

where ∇_μ denotes the covariant derivative on the four-dimensional smooth pseudo-Riemannian manifold V^4 and $[A_\mu \cdot]$ denotes the commutator. The derivative D_μ acts on φ as follows

$$(D_\mu \varphi)^a = \nabla_\mu \varphi^a + g f^{abc} A^b_\mu \varphi^c; \tag{3}$$

here $\nabla_\mu \varphi^a = \partial_\mu \varphi^a$, of course. The Einstein-Yang-Mills-Klein-Gordon (EYMKG) equations are

$$R_{\mu\nu} - \frac{g_{\mu\nu}}{2} R = T_{\mu\nu}(\mathrm{YM}) + T_{\mu\nu}(\mathrm{KG}), \tag{4}$$

$$D_i F^{i0} = -j^0, \tag{5a}$$

$$D_\mu F^{\mu k} = -j^k, \tag{5b}$$

$$D_i D^i \varphi + D_0 D^0 \varphi - \frac{\partial V}{\partial \varphi} = 0, \tag{6}$$

where

$$j^a_\mu = gf^{abc}\varphi^b(D_\mu\varphi)^c, \qquad T_{\mu\nu}(\mathrm{KG}) = -F^a_{\mu\gamma}F^{a\gamma}_{\nu\cdot} + \tfrac{1}{4}\, g_{\mu\nu}F^a_{\gamma\delta}F^{a\gamma\delta},$$

$$T_{\mu\nu}(\mathrm{KG}) = (D_\mu\varphi)^a(D_\nu\varphi)^a - \frac{g_{\mu\nu}}{2}\left[(D_\alpha\varphi)^a(D^\alpha\varphi)^a + V(\varphi)\right].$$

The term $V(\varphi)$ describes selfinteraction of the scalar field, except for the case when it is a bilinear or linear function of its argument.

For a stationary manifold V^4 there exists a system of local coordinates in which the metric tensor is time-independent and $g^{00} > 0$; if in addition, $g^{0i} = 0$ then the manifold is said to be static. Let W_3 be the space-oriented section (x_0 = const) of such a system. Then in W_3 one can choose a negatively definite metric tensor g_{ij} and for every tangent vector $\frac{dx^u}{ds}$ we have $g_{ij}\frac{dx^i}{ds}\frac{dx^j}{ds} < 0$ (denotation is as in Weder's paper [4]).

We will look for nontrivial solutions of Eqs. (3)–(5), i.e., solutions with nonvanishing curvatures and nonzero φ. It will be assumed that the space sections are noncompact and that fields satisfy the following asymptotic conditions

$$g_{00} = 1 - \frac{M}{r} + o\left(\frac{1}{r}\right), \qquad |\partial_k g_{00}| = \frac{-M}{r^2} + o\left(\frac{1}{r^2}\right) \quad (M \text{ is either zero or negative}),$$

$$g_{ik} = O\left(\frac{1}{r}\right) + \eta_{ik}, \qquad |\partial_l g_{ik}| = O\left(\frac{1}{r^2}\right), \tag{7}$$

$$|A^a_\mu| \leqslant \frac{c}{r^{\frac{1}{2}+\varepsilon}}, \qquad |\partial_k A^a_\mu| \leqslant \frac{c}{r^{\frac{3}{2}+\varepsilon}}, \tag{8a}$$

$$|\varphi| \leqslant \frac{c_0}{r^{\frac{1}{2}+\varepsilon}}, \qquad |\partial_k\varphi| \leqslant \frac{c_0}{r^{\frac{3}{2}+\varepsilon}}. \tag{8b}$$

These conditions ensure that the energy of the system is finite and all integrals appearing below are also finite.

Remark 1. We suppose, following Weder, that at infinity the time component of a metric tensor g_{00} tends to unity like $1/r^{1+\varepsilon}$ (note: everything which is to be said below refers also to the case with nonzero negative M). It is likely that under this requirement our result (as well as Weder's) follows from the positive energy theorems[1] [7]; this is the point of view of Deser [5]. To check this one has to prove that the above condition on g_{00} forces also g_{ij} to tend to 1 like $1/r^{1+\varepsilon}$, implying the gravity mass to vanish. The absence of nonzero energy solutions is then an immediate consequence of the mass positivity theorem. The author does not see a simple explanation of the relation between asymptotics of g_{00} and g_{ij}, although it could exist; note the asymptotic behaviour of the Schwarzschild metric,

[1] The author is grateful to Professor A. Staruszkiewicz for pointing out this fact and for several illuminating discussions.

for instance. In any case, the calculations which are to be made, considerably simplify our system of equations. The simplified equations will be used in Section 3, where we will weaken the conditions imposed on the metric.

Now we are going to establish:

Theorem 1. EYMKG system of equations does not possess static smooth solutions with nonvanishing curvatures satisfying conditions (7), (8) if the following inequality holds

$$\int\limits_{W_3} \varphi^a \frac{\partial V}{\partial \varphi^a} \leqslant 0. \tag{9}$$

Proof. Below we will show that a scalar field $\{\varphi^a\}$ vanishes, provided that the foregoing inequality is satisfied. Then our equations reduce to the Einstein-Yang-Mills system and the above proposition is the consequence of Weder's results [4]. The equations to be used are Gauss-like equations of Yang-Mills theory

$$D_i F^{ai0} = -g^2 f^{abc} \varphi^b f^{cdf} A^{d0} \varphi^f \tag{10}$$

and the Klein-Gordon static equation

$$(D_i D^i)^a + g^2 f^{abc} A_0^b f^{cdf} A^{d0} - \frac{\partial V}{\partial \varphi^a} = 0. \tag{11}$$

Here the right hand side of (10) stands for the zeroth component of the Klein-Gordon current. Let us multiply Eqs. (10) and (11) by A_0^a and φ^a respectively. Then, integrating by parts and omitting boundary terms, one comes to the system

$$-\int\limits_{W_3} g_{ik} F^{ai0} F^{ak0} dV_3 = -g^2 \int\limits_{W_3} (f^{abc} A^{a0} \varphi^b)(f^{cdf} A^{d0} \varphi^f) dV_3, \tag{12}$$

$$\int\limits_{W_3} \left[-g_{ik}(D^i\varphi)^a (D^k\varphi)^a - \varphi^a \frac{\partial V}{\partial \varphi^a} \right] dV_3 = g^2 \int\limits_{W_3} (f^{abc} A_0^a \varphi^c)(f^{cdf} A^{d0} \varphi^f) dV_3. \tag{13}$$

Notice that the metric g_{ij} is negative definite on W_3, $g^{00} > 0$ and $g_{0i} = 0$; hence one concludes that the l.h.s. of (12) is weakly positive, while its r.h.s. is nonpositive. It follows that both sides of (12), and consequently of (13) (since the r.h.s. of (13) is equal to the r.h.s. of (12) with opposite sign), should vanish. Therefore the curvatures F_{0i} and the covariant derivative $D_i\varphi$ (because of (9)) also disappear. Let us analyze further the equation

$$(D_i\varphi)^a = \partial_i \varphi^a + g f^{abc} A_i^b \varphi^c = 0. \tag{14}$$

Let us multiply (14) by φ^a and sum over the free index a. Then we get

$$\varphi^a \partial_i \varphi^a = \partial_i(\varphi^a \varphi^a/2) = -g f^{abc} A_i^b \varphi^d \varphi^c = 0. \tag{15}$$

Since at infinity the scalar field has to vanish, one concludes that $\varphi = 0$. Now the proposition follows directly from Weder's results [4], because, as it was stated at the beginning of the proof, the equations reduce to the Einstein-Yang-Mills system.

A simple example of $V(\varphi)$ satisfying (9) is given by $\frac{\partial V}{\partial \varphi^a} = F(|\varphi|)\varphi^a$ where $F \leqslant 0$ and $|F(x)| \leqslant$ const as $x \to 0$. This last condition is imposed in order to guarantee finiteness of the l.h.s. of (13). It may be relaxed if the scalar field decreases faster at infinity, than it was assumed in (8b).

As a direct consequence of the preceding theorem we get

Corollary. The EYMKG equations have no nontrivial static solutions satisfying boundary conditions (7), (8), when the scalar field is linear.

Proof. It suffices to check that $\varphi^a \frac{\partial V}{\partial \varphi^a} = -m^2\varphi^a\varphi^a \leqslant 0$, where m is the real mass; thus (9) is satisfied.

3. *Absence of small gauge fields*

In the preceding Section we proved vanishing of the linear scalar field and of the time component of Yang-Mills potential. That result does not depend on the specific behaviour of a metric at infinity; the required falloff could be arbitrary.

Let us consider the Yang-Mills equations for remaining components of a potential. They read

$$\frac{1}{\sqrt{-g}}\partial_i(\sqrt{-g}\, g^{il}\partial_l)A^{ak} - \partial^k\partial_i A^{ai}$$
$$= -f^{abc}A^{bi}\partial_i A^{ck} - \Gamma^{\mu}_{i\mu}(F^{aik} - \partial_i A^{ak}) - f^{abc}A^b_i F^{cik}. \tag{16}$$

This system could be made elliptic supposing (as we do now) that divergence $\partial_i A^{ai}$ vanishes. We expect that its solutions are at least twice differentiable: as a matter of fact it could be shown even more, that they are of class C^∞. The proof (which needs a sequence of Sobolev regularity theorems) is left to the interested reader. Thus it makes sense to demand that our solution A^a_i belongs to the Sobolev space $W_{1,3}$. Let us remind that Sobolev spaces $W_{1,k}$ are defined as the closure of smooth functions with the norm $||\varphi||_{W_{1,k}} = [\int dV(|\varphi|^k + (\partial\varphi)^k)]^{1/2}$. In addition we need $|\partial_i A^a_k| \in L_2$, $|A^a_k| \in L_4 \cap L_6$, $|\Gamma^{\mu}_{i\mu}| \in L_3$ (here $\Gamma^{\mu}_{i\mu}$ is the contracted Christoffel symbol).

One could define also covariant Sobolev norms, for instance

$$||\varphi||_{1,2\mathrm{cov}} = [\int dV_3(\varphi^2 + g^{ik}\partial_i\varphi\partial_k\varphi)]^{1/2}.$$

I make the conjecture that in our case the covariant $W_{p,k}(L_p)$ norms are equivalent to the usual $W_{p,k}(L_p)$ norms (see Appendix for a discussion). The following estimate

$$\int \varphi^6 dV \leqslant c_1[\int (\partial\varphi)^2 dV]^3 \tag{17}$$

will be useful below. It is known in Euclidean space [8] but it could be extended on Riemannian manifolds as well [7] if the mean extrinsic curvature of W_3 vanishes. Now we can estimate the curvature F

$$\|F\|_{L_2} \equiv [\sum_{a,i,j} \int dV(F^a_{ij})^2]^{1/2} \leqslant \sum_{a,i,j} [\int dV(\partial_i A^a_j)^2]^{1/2}$$

$$+ \sum_{\substack{a,b,c\\ i,j,k}} (\int dV f^{abc} f^{ade} A^b_i A^c_j A^{di} A^{ej})^{1/2} \leqslant c_2[\|\partial A\|_{L_2} + \sum_{a,i} (\int dV(A^a_i)^4)^{1/4}]$$

$$\leqslant c_3 \|\partial A\|_{L_2}(1 + \|A\|_{L_3}). \tag{18}$$

The first inequality is Minkowski while the second follows from the Hölder inequality. The last one is yielded by (17) (see (21) below). Now we prove the following result.

Theorem 2. Let the falloff of A^a_i, $\partial_k A^a_i$ at infinity be $A^a_i = O\left(\frac{1}{r^{1+\varepsilon}}\right)$, $\partial_k A^a_i = O\left(\frac{1}{r^2}\right)$ respectively. Suppose $\Gamma^\mu_{i\mu} \in L_3$ and let $|g_{\mu\nu} - \eta_{\mu\nu}| = O\left(\frac{1}{r}\right)$ at infinity. Then the system (16) does not possess solutions A^a_i small in $W_{1,3}$ norm provided that the L_3 norm of $\Gamma^\mu_{i\mu}$ is sufficiently small.

Proof. Let us multiply (16) by A^a_k. Integration by parts and omission of boundary terms yields

$$\|\partial A\|^2_{L_2} = -\int f^{abc} A^a_k A^{bi} \partial_i A^{ck} dV - \int \Gamma^\mu_{i\mu} A^a_k (F^{aik} - \partial^i A^{ak}) dV - \int f^{abc} A^a_k A^b_i F^{cik} dV. \tag{19}$$

Here and elsewhere below we use the equivalence of covariant and usual L_p norms. Using the Minkowski and Hölder inequalities one obtains

$$\|\partial A\|^2_{L_2} \leqslant c_4[\|A\|^2_{L_4}\|\partial A\|_{L_2} + \|\Gamma\|_{L_3}\|A\|_{L_6}(\|F\|_{L_2} + \|\partial A\|_{L_2}) + \|A\|^2_{L_4}\|F\|_{L_2}], \tag{20}$$

where $\|\Gamma\|_{L_3} = \sum_i \|\Gamma_i{}^\mu_\mu\|_{L_3}$. Hölder inequalities and (17) yield an estimation on $\|A\|^2_{L_4}$

$$\|A\|^2_{L_4} = (\sum_{a,i} \int (A^a_i)^4 dV)^{1/2} \leqslant [\sum_{a,i} (\int |A^a_i|^3 dV)^{2/3} (\int (A^a_i)^6 dV)^{1/3}]^{1/2}$$

$$\leqslant c_5 \|A\|_{L_3} \|\partial A\|_{L_2}. \tag{21}$$

Using the above estimates on $\|F\|_{L_2}$, $\|A\|_{L_6}$ and $\|A\|_{L_4}$ in (20) results in the following inequality

$$|\partial A\|^2_{L_2} \leqslant c_6 \|\partial A\|^2_{L_2}(\|A\|_{L_3} + \|\Gamma\|_{L_3}(1 + \|A\|_{L_3}) + \|A\|_{L_3}(1 + \|A\|_{L_3})). \tag{22}$$

Taking into account that

$$\|A\|_{L_3} \leqslant \|A\|_{W_{1,3}}, \qquad \|\partial A\|_{L_3} \leqslant \|A\|_{W_{1,3}} \tag{23}$$

we eventually come at a crude estimation:

$$\|\partial A\|^2_{L_2} \leqslant c_7 \|\partial A\|^2_{L_2}(\|A\|_{W_{1,3}}(2 + \|A\|_{W_{1,3}}) + \|\Gamma\|(1 + \|A\|_{W_{1,3}})). \tag{24}$$

Supposing $||A||_{W_{1,3}} < \frac{1}{6c_7} < 1$, $||\Gamma|| < \frac{1}{2c_7\left(1+\frac{1}{6c_7}\right)}$ we obtain a contradiction

$$||\partial A||_{L_2}^2 \leqslant a||\partial A||_{L_2}^2, \quad a < 1; \tag{25}$$

it implies the absence of (sufficiently small in $W_{1,3}$ norm) solutions of equations (16).
Remark 2. The assumption that $\Gamma_{i\mu}^{\mu}$ is small in L_3 norm does not mean that the gravity is weak. In the latter case one could prove absence of Yang-Mills fields which are not necessary cubic integrable, as it was supposed above. Indeed, let $A_i^a = O\left(\frac{1}{r}\right)$ and $F_{ik}^a = O\left(\frac{1}{r^2}\right)$. Then we have

$$0 = \int_{R^3} \partial_i(\sqrt{-g}\, x_k T^{ki}) d^3x = \int_{R^3} \sqrt{-g}\,(T_i^i - x_k \Gamma_{\alpha\beta}^k T^{\alpha\beta}) d^3x$$
$$= \int \sqrt{-g}\,(T_0^0 - x_k \Gamma_{ij}^k T^{ij} - x_k \Gamma_{00}^k T^{00}) d^3x. \tag{26}$$

Here we used the conservation of the energy-momentum tensor. Rewriting (26), using estimations on the metric tensor and on its derivatives and estimating T_{ij} by T_{00} we have

$$\int \sqrt{-g}\, T_0^0 d^3x \leqslant c_8 \sup_{x\in R^3} \{|x_i \Gamma_{\alpha\beta}^i|\} \int \sqrt{-g}\, T_0^0 d^3x. \tag{27}$$

For sufficiently weak gravitational fields one concludes that both,sides of the inequality have to vanish; thus $F_{ik}^a = 0$.
Corollary. Let the nonabelian fields be sufficiently small in $W_{1,3}$ norm and let $\Gamma_{i\mu}^{\mu}$ be small in L_3. Then the Einstein-Yang-Mills system reduces to the sourceless Einstein equations and the 3-space if flat.
Proof. It remains to prove only the last assertion. Now the Einstein equations read

$$R_{\mu\nu} = 0. \tag{28}$$

From $R^{00} = 0$ it follows $g^{00} = 1$ and therefore the vanishing of the Ricci tensor is equivalent to the vanishing of the Ricci tensor of the three dimensional manifold W_3. It implies flatness of W_3.

4. The local analysis of the existence problem

In practice the criterion (9) is often useless. For instance there is an important class of scalar field selfinteractions given by $\frac{\partial V}{\partial \varphi^a} = \lambda(\varphi^b\varphi^b - \alpha^2)\varphi^a$; λ, α — reals, for which it is not possible to determine the sign of $\lambda \int_{W_3} \varphi^a\varphi^a(\varphi^b\varphi^b - \alpha^2) dV_3$ without knowing a solution. So the Theorem 1, global in its nature, is now not applicable. One can use, however, certain local techniques taken from bifurcation theory.

Let us describe briefly the procedure that can be used here. At the first stage there should be chosen appropriate functional spaces, satisfying those boundary conditions, which are implied by physics. They should be also suitable from the technical point of view. On compact manifolds the most convenient are the Sobolev or Hölder functional spaces. On noncompact manifolds, which is the case here, appropriate are spaces of Nirenberg--Walker and Cantor [9]. At the second step the equations should be linearized around the solution which is known explicitly. The EYMKG system has one exact solution

$$\{g^{\mu\nu}\} = \begin{pmatrix} 1 & & & \\ & -1 & & 1 \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}, \qquad A^a_\mu = 0, \qquad \varphi^a = 0. \tag{29}$$

Linearization of (4–6) at (29) results in the following equations

$$\delta R = 0, \tag{30a}$$

$$\Delta \delta A^a_\mu = 0, \tag{30b}$$

$$\Delta \delta \varphi^a = -\lambda \alpha^2 \varphi^a. \tag{30c}$$

Here δR denotes the linearization of the Ricci scalar R and the r.h.s. of (30c) follows from the linearization of $\dfrac{\partial V}{\partial \varphi^a}$, with $\dfrac{\partial V}{\partial \varphi^a}$ being specified above. The product $\sigma = \lambda\alpha^2$ can now be treated as an eigenvalue of the operator defined by the l.h.s. of (17). In most known versions of bifurcation theory the eigenvalues are supposed to be isolated. If it is so, then the existence of eigenfunctions at some eigenvalue (say, $\sigma_0 = \lambda\alpha^2$) is a hint for bifurcation, i.e. for the occurrence of those solutions of EYMKG system, which at $\sigma = \sigma_0$ coincide with the trivial one, and for $\sigma \approx \sigma_0$ might be given by a series expansion. On bounded manifolds with a boundary the eigenvalues are indeed isolated and then it follows from some versions of the Krasnoselsky theorem that if an eigenvalue is of odd multiplicity, there should appear bifurcating solutions. For the relevant information see [8] and [10]. In our case, however, the spectrum is continuous. There is a version due to Stuart [11] that could be adapted to this instance. But it is out of the scope of the paper.

Remark 3. Recently the strong coupling ($g \gg 1$) regime in Yang-Mills theory with sources has been investigated [12]. The $1/g$ expansion of nonabelian solutions was proposed in the $g \to \infty$ limit. This formalism can be extended also to scalar fields coupled to gauge fields. It seems to be interesting to study the existence of solutions in this limit for the scalar fields of Higgs type.

APPENDIX

We would like to show the equivalence of usual and covariant L_p norms, that is we wish to prove the existence of two positive numbers a, b such that

$$a\|\cdot\|_{L_p} \leqslant \|\cdot\|_{L_p,\mathrm{cov}} \leqslant b\|\cdot\|_{L_p}. \tag{A1}$$

Let us consider

$$\|\varphi\|_{L_p,\mathrm{cov.}} = \left(\int d^3x\,\sqrt{-g}\,(g^{ik}\partial_i\varphi\partial_k\varphi)^{p/2}\right)^{1/p}. \tag{A2}$$

Because of boundary conditions $|g_{\mu\nu}-\eta_{\mu\nu}| = O(1/r)$ we conclude that outside a sphere with a sufficiently large radius R following inequalities hold: $\sqrt{-g}|g_{\mu\nu}| < 2$ and $\sqrt{-g}|g_{\mu\nu}| > 1/2$. The $g_{\mu\nu}$'s are supposed to be smooth, therefore inside the sphere they attain their maximal (minimal) values; let K_1 be such that $|g_{\mu\nu}| < K_1$ for $r \leqslant R$. Similarly, there exists a finite K_2 such that $\sqrt{-g} < K_2$ inside the sphere. Thus let $b = \max\{2, K_1 \times K_2\}$.

For physical reasons the eigenvalues h_μ of $\{g_{\mu\nu}\}$ should be nonzero; let $a = \min\{1/2, \frac{1}{4}\sqrt{-g}\{|h_0|, |h_1|, |h_2|, |h_3|\}; r \leqslant R\}$. Then we arrive at (A1).

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